

COMPLETELY COLORBLIND: ADVANCES IN GRAY TECHNIQUES AND APPLICATIONS TO PLANETS NEAR AND FAR.

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Introduction and History

Gray approaches, which replace spectrally-resolved opacities with a wavelength independent mean opacity, have seen countless applications to problems of radiative transfer in stellar and planetary atmospheres since the original work of Schwarzschild (1906). Applying the radiative equilibrium expressions of Schwarzschild (1906) to planetary thermal structure requires the assumption of an atmosphere that is essentially transparent to solar radiation, such that all shortwave absorption occurs at the surface or, for gas giants, at infinite depth. These models, with different opacities for shortwave versus long-wave radiation, are referred to as semi-gray, although the prefix is often dropped in the planetary literature.

Of course, planetary atmospheres are not transparent to shortwave radiation. For Earth, water vapor is the main atmospheric absorber of solar radiation, and, in this context, Emden (1913) and Milne (1922) were the first to explore how shortwave attenuation influences the radiative equilibrium thermal structure of an Earth-like atmosphere. Later, Hopf (1934) and Wildt (1966) explored the general solution to the semi-gray radiative equilibrium problem where both shortwave and long-wave fluxes are attenuated.

McKay et al. (1999) linearly combined the analytic model of Schwarzschild (1906) with those of Emden (1913) and Milne (1922) to produce radiative equilibrium thermal structure profiles with both shortwave heating at the surface and in the upper atmosphere. These models could develop stratospheric inversions, and were a very good match to the observed temperature structure of Titan's atmosphere, which is mostly in a state of radiative equilibrium. Building on these results, Robinson and Catling (2012) derived the first generalized analytic radiative-convective model for planetary atmospheres. This model is semi-gray, with two shortwave channels—one for heating the deep atmosphere and one for heating the upper atmosphere—and self-consistently determines where the radiative solution transitions to a convective adiabat. Figure 1 shows the evolution of gray models, from Schwarzschild (1906) to Robinson and Catling (2012), as applied to Jupiter.

Today, the explosion of results and findings in the field of exoplanetary science has led to a resurgence of gray models for simulating planetary atmospheric structure. Since exoplanet atmospheric characterization is, by and large, observationally limited, simple gray approaches can be much better constrained than complex, multi-parameter models. As is discussed below, not only

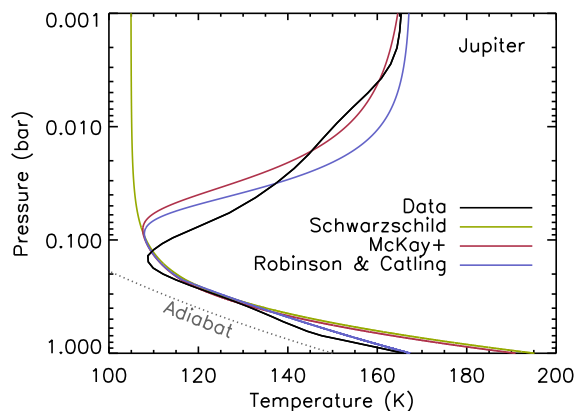


Figure 1: Applications of the Schwarzschild (1906), McKay et al. (1999), and Robinson and Catling (2012) models to Jupiter.

are gray techniques useful for providing constraints on atmospheric characteristics of exoplanets, but they also enable comparative studies of planets near and far by capturing the essential physics of planetary atmospheres in models that are broadly applicable and straightforward to interpret.

Two Gray Approaches

Gray techniques have been applied in both dynamic and equilibrium models. For the former, pressure (p) dependent profiles of temperature (T) and atmospheric composition are used to determine the gray opacities of model levels at some timestep (using, e.g., a lookup table). These opacities give the relationship between pressure and optical depth, and, thus, can be used when solving the two-stream equations of radiative transport (Schuster 1905; Schwarzschild 1906). Gradients in the net radiative flux then drive atmospheric heating and cooling, which are used to update the T - p profile as the model timesteps forward.

Equilibrium models take a different approach. Here, the two-stream equations are combined with an assumption of radiative (Schwarzschild 1906; Emden 1913; McKay et al. 1999) or radiative-convective (Robinson and Catling 2012) equilibrium to derive an analytic τ - T profile. Given a p - τ relationship, either from an assumed parameterization or computed numerically, the equilibrium T - p profile can be determined. A commonly assumed relationship between optical depth and pressure

is a power law, with $p \propto \tau^n$ (Pollack 1969). As shall be discussed in the next section, the power n has certain physical interpretations.

Gray Opacities

Two key approaches to determining gray opacities exist. First, gray opacities can be computed using spectrally-resolved gas (or grain) absorption spectra, commonly generated using line list databases (e.g., HITRAN; Rothman et al. 1987, 2013). Such calculations are generally performed over a grid of pressure (p) and temperature (T) points, and assume some atmospheric chemical composition (from, e.g., thermal equilibrium chemistry). The result is a lookup table of pressure and temperature dependent gray opacities. It is uncommon to see gray opacities tabulated for terrestrial planetary atmospheres, as one cannot generally make the assumption that the atmosphere is a solar (or metal-enriched solar) composition gas in thermo-chemical equilibrium (which is typically used when computing gas giant opacities). Thus, especially for terrestrial planets, it is common to heed the advice of Thomas and Stamnes (1999) who suggest determining gray opacities via comparisons between gray models and observations. In this second key approach, one simply uses gray opacities as fitting parameters or as tools for comparative climatology.

When using spectrally-resolved models to compute gray opacities, a weighting is usually applied while integrating the resolved opacities (κ_ν) over frequency. One common averaging technique is the “Planck mean”, where the mean opacity is determined using a weighting of the form $\kappa_\nu B_\nu(T)$, where B_ν is the Planck function. This weighting emphasizes spectral regions near the peak of the Planck function where the resolved opacity is high, thereby ensuring that the gray flux *emitted* by a thin atmospheric slab agrees with spectrally-resolved models. A second averaging technique, called the “Rosseland mean”, emphasizes regions near the peak of the Planck function where the resolved opacity is low, and uses a weighting of the form $\kappa_\nu^{-1} dB_\nu(T)/d\nu$. This particular weighting ensures that the radiation diffusion limit is obeyed, thus making the Rosseland mean an appropriate choice deep in a planetary atmosphere (Mihalas 1970).

Recent tabulations of Planck and Rosseland mean opacities for brown dwarf and giant planet atmospheres can be found in Freedman et al. (2008, 2014). Figure 2 shows examples of gray column optical depths through an isothermal Jupiter-like atmosphere for the Freedman et al. opacities as well as a profile for Titan (from McKay et al. 1999). Notice that the Freedman et al. Rosseland mean optical depths, which are most sensitive to continuum collision-induced absorption (CIA) as well as Lorentzian line wings and apply best in the deep atmosphere, show a $\tau \propto p^2$ scaling. Both CIA

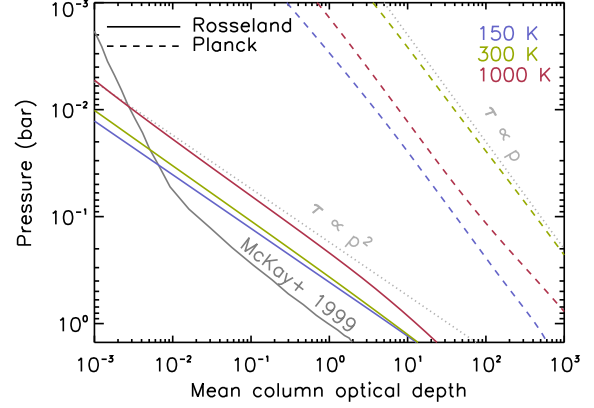


Figure 2: A selection of mean optical depths from the literature. Power laws with $\tau \propto p$ and $\tau \propto p^2$ are shown. See text for more details.

and collisional line broadening are pressure dependent processes, such that one would expect $d\tau \propto p dp$ (Sagan 1969), as the gray opacity (κ) will be roughly proportional to pressure. For the Freedman et al. Planck mean opacities, which are most sensitive to Doppler broadened line cores and apply best in the upper atmosphere, we see a $\tau \propto p$ scaling. This weaker dependence on pressure is due to the lack of pressure dependence in the Maxwellian distribution of molecular velocities, giving κ roughly independent of p . Note that $\tau \propto p$ and $\tau \propto p^2$ scalings are seen in the McKay et al. Titan gray optical depth profile in the upper and lower atmosphere, respectively.

The analysis above would indicate that power law indexes for the $p \propto \tau^n$ relationship of roughly $n = 1-2$ are most appropriate. Indeed, McKay et al. (1999) found that a value of $n = 4/3$ produced a good fit of their gray thermal structure models to Titan’s observed T - p profile. Note, however, that if the species primarily responsible for providing thermal opacity has a mixing ratio profile that varies with pressure, then larger or smaller values of n might be expected. For example, Weaver and Ramanathan (1995), when considering the small scale height for water vapor in Earth’s atmosphere, suggest $n = 4$. However, Robinson and Catling (2014; their Figure S4) showed that using $n = 2$ for models of gray thermal radiation in Earth’s troposphere best reproduces thermal flux profiles from spectrally resolved models.

Gray in 3-D

The computational efficiency of gray radiative transfer has led to its adoption in numerous 3-D models of atmospheric circulation. For example, in their studies of methane storms on Titan using 3-D models, Mitchell et al. (2011) use a gray radiative transfer scheme with

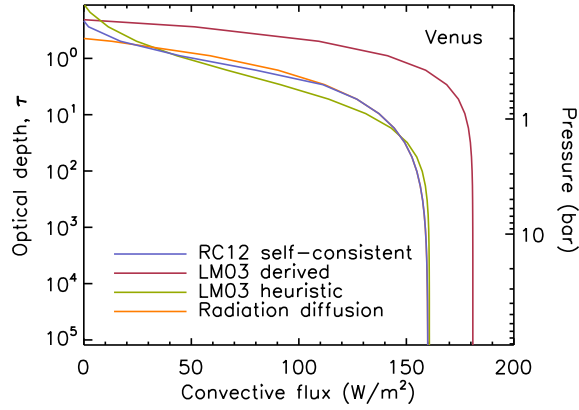


Figure 3: Convective flux in the Cytherean atmosphere from several gray approaches. See text for more details.

heritage in the McKay et al. (1999) models. Note that Mitchell et al. assume a power law index of $n = 3/2$, which is weighted more towards the deep atmosphere than the $n = 4/3$ value used by McKay et al. throughout the entire troposphere and stratosphere.

Frierson et al. (2005) also adopt a gray radiation scheme in their simulations of moist processes in a 3-D Earth-like model. Here, however, to more properly represent the dependence of opacity on pressure in both the troposphere and stratosphere, Frierson et al. assume $\tau \sim f(p/p_0) + (1 - f)(p/p_0)^4$, where the parameter f controls the transition from opacities being primarily dominated by water vapor and pressure broadening to being dominated by Doppler broadening, and p_0 is a reference pressure (at, e.g., the surface). Of course, an added bonus from this parameterization is that radiative relaxation timescales in the stratosphere are much shorter, which can decrease model spin-up time. More recently, Heng et al. (2011) generalized this parameterization and applied it to 3-D studies of atmospheric circulation in Hot Jupiter atmospheres.

Gray Comparative Climatology

The generality of gray techniques make them ideally suited to studies in comparative climatology and planetary climatology. For example, after developing an analytic semi-gray model of Titan's thermal structure and associated haze effects, McKay et al. (1999) then used their anti-greenhouse models to study surface temperatures of a hazy early Earth. More recently, Robinson and Catling (2014) used an analytic semi-gray radiative-convective model to explain the common ~ 0.1 bar temperature minimum in the thermal structure of Earth, Jupiter, Saturn, Titan, Uranus, and Neptune.

A common theme in comparative climatology works that use gray techniques is that of convection. Sagan (1969) used gray and windowed-gray models to ana-

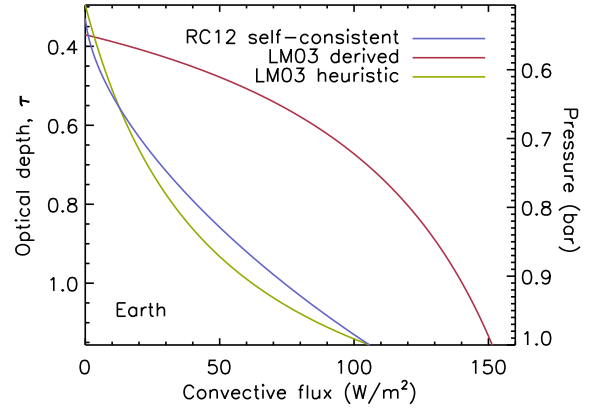


Figure 4: Convective flux in Earth's atmosphere from several gray approaches. See text for more details.

lyze the criteria for the onset of convective instability, which was revisited by Weaver and Ramanathan (1995) with an emphasis on the role that the vertical distribution of greenhouse gases plays in setting the steepness (and, thus, stability) of a radiative equilibrium temperature profile. Lorenz and McKay (2003) used the Schwarzschild (1906) gray gas model to arrive at analytic expressions for the convective flux in a planetary atmosphere that is transparent to shortwave radiation.

Figures 3 and 4 show profiles of convective flux through the atmospheres of Venus and Earth, respectively, for several different models. All models assume $n = 2$ (which is appropriate for our focus on the deep atmospheres of these worlds), that all shortwave absorption occurs at the surface, and that the temperature profile in the convective portion of the atmosphere follows a dry adiabat. The Robinson and Catling (2012; RC12) models are a self-consistent solution to the radiative-convective equilibrium problem, and adjoin a radiative equilibrium thermal structure profile to a convective profile by ensuring continuity of thermal radiative flux and temperature across the radiative-convective boundary. The Lorenz and McKay (2003; LM03) “derived” result has a convective flux profile that follows $F_c = F_s[1.5 - \frac{R_s}{c_p} \frac{4}{nD\tau}(1 + D\tau)]$, where F_s is the solar flux absorbed at the surface, R_s is the specific gas constant, c_p is the atmospheric specific heat capacity, and D is the so-called diffusivity factor that accounts for the integration of intensities over a hemisphere to arrive at the fluxes used in two-stream approaches. We also show the Lorenz and McKay (2003) “heuristic” form of the convective flux, with $F_c = F_s(\tau - \tau_{rc})/[A + B(\tau - \tau_{rc})]$, where A and B are fitting parameters. Note that we have adjusted this expression slightly to ensure that the convective flux goes to zero at the radiative-convective boundary (located at τ_{rc}). Figure 3 also shows the analytic result for the radiation diffusion limit, where the net thermal flux is $\frac{2}{D} \frac{d\sigma T^4}{d\tau}$, so that (for an adiabatic tem-

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perature profile and with $\tau \propto p^n$) the convective flux is

$$F_c = F_s - \frac{8R_s \sigma T_0^4}{n c_p D \tau_0} \left(\frac{\tau}{\tau_0} \right)^{\frac{4R_s}{n c_p} - 1}, \quad (1)$$

where T_0 is the temperature at the surface (located at τ_0).

The Venus and Earth cases both demonstrate that the derived result from Lorenz and McKay (2003) is not particularly good representations of the convective flux. This likely stems from conflicting assumptions used when arriving at their expression—the temperature gradient is taken as adiabatic while the temperature profile is assumed to be in radiative equilibrium. The Lorenz and McKay (2003) heuristic model, which we fit to the self-consistent profile, yields much better convective flux profiles. The functional form of this model is able to reproduce profiles ranging from the extremely optically thick Venus case to the relatively optically thin Earth case.

Conclusions and Future Work

The simplicity of gray models makes them ideal for gaining intuition, while their generality makes these tools ideal for application to a broad range of planetary conditions. It is no wonder that gray techniques and their application have been an active area of research for over 100 years.

A particularly exciting application of gray techniques that is currently seeing active development is in exoplanet retrieval analysis. Here, gray radiative equilibrium thermal structure models are used to specify an atmospheric temperature profile with as few as four parameters (e.g., Line et al. 2012), and a radiative-convective equilibrium thermal structure could be specified with as few as 5–6 parameters. This approach is especially attractive when attempting to minimize the number of parameters in a retrieval analysis, and offers an advantage over fitting for many tens of level dependent temperatures.

While gray models are currently seeing wide and diverse applications, a well known deficiency remains—gray radiative equilibrium profiles tend towards a constant “skin temperature” at low pressures. This behavior is obviously unphysical, as was recently highlighted in a short paper on moist greenhouse atmospheres by Kasting et al. (2015). However, recent progress in developing gray-like models with “picket fence” opacities (Parmentier et al. 2015) have improved fits to atmospheric thermal structures at low pressures. Building on these results will enable the application of gray techniques to an even broader range of atmospheric conditions.

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